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ANALYSES OF MEAN AND TURBULENT MOTION IN THE TROPICS WITH THE USE OF UNEQUALLY SPACED DATA

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SUMMARY

Wind velocities from 25 km to 60 km over Ascension Island, Fort Sherman and Kwajalein for the period January 1970 to December 1971 are analyzed in order to achieve a better understanding of the mean flow, the eddy kinetic energy and the Eulerian time spectra of the eddy kinetic energy.

Since the data is unequally spaced in time, techniques of one-dimensional covariance theory are utilized and an unequally spaced time series analysis is accomplished. The problems in the analysis of unequally spaced time series data are discussed and the theoretical equations for two-dimensional analysis or wavenumber frequency analysis of unequally spaced data are developed.

Analysis of the turbulent winds and the average seasonal variance and eddy kinetic energy of the turbulent winds indicates the maximum total variance and energy is associated with the east-west velocity component. This is particularly true for the long period seasonal waves which dominate the total energy spectrum. Additionally, there is an energy shift for the east-west component into the longer period waves with altitude increasing from 30 km to 50 km.

INTRODUCTION

There have been many studies of the means and energy spectrums of atmospheric turbulence and diffusion for the variables in the free atmosphere, particularly below the tropopause. While studies of the mean and seasonal variations of atmospheric variables have been extended into the stratosphere and mesosphere, almost all have used extensive interpolation. Due to the lack of data and the limited reports from a very few reporting stations, little work has been accomplished on the structure of turbulence in the stratosphere and mesosphere.

Studies of the vertical power spectra of winds over White Sands, New Mexico, (Kao and Sands, 1966) and the Eulerian time spectra of winds over Cape Kennedy, Florida, (Norton, 1967) represent some of the earliest spectral work in the stratosphere and lower mesosphere. Since then, several persons have made estimates of the structure of turbulence for winds, temperature, pressure and density, (Hirota, 1975) being one of the latest. Many others have used structure functions or differencing techniques to estimate the amplitude and phase of different types of atmospheric waves; Justis and Woodrum (1972, 1973), Belmont et al. (1974) and Nastrom and Belmont (1976) are a few. However, without exception, all of these studies required interpolation of the data to equally spaced intervals, and in some of the studies more than half of the data points were interpolated.

Recent studies by Julian and Cline (1974), Jones (1975) and Julian and Thiebaux (1975) have dealt with the estimation of the spacial wave-number spectra using unequally spaced data observation points. If it is assumed a process is circular and statistically stationary, the following relationship holds:

$$\sum_{k=-\infty}^{\infty} Z_k \exp(i\ 2\pi\ k\ D_{i,j}) = E[x(\lambda_i)\ \overline{x(\lambda_j)}]$$

where k is the normalized wavenumber, Z_k is the complex coefficient of amplitude, 2π represents the unit circle, $D_{i,j}$ is the normalized distance over the unit circle between points λ_i and λ_j, E is the expected value function, $x(\lambda_i)$ is a realization of a process x at point λ_i, and $\overline{(\ )}$ is the complex conjugate. Further, under the same assumptions this relationship is extended to time series by the following equation:

$$\sum_{n=-\infty}^{\infty} S_n \exp(i\ 2\pi\ n\ T_{i,j}) = E[f(T_i)\ \overline{f(T_j)}]$$

Here n represents a form of normalized frequency (the number of waves over a fixed normalizing period), the sign of n represents the direction of propagation, $T_{i,j}$ is the normalized time between T_i and T_j and $f(T_i)$ is a realization of the process at time T_i.

The purpose of this study is to first determine general features of the mean and turbulent wind flow in the tropics for the east-west and north-south components of velocity. No interpolated data points will be used in the analysis except to detrend and smooth the data. Next, the study will determine the general features of the normalized Eulerian time spectra by season for synoptic scale and longer period waves. In analyzing the spectrum, unequally spaced time series analysis will be used. Finally, this study will discuss and develop the general equations of unequally spaced wavenumber-frequency analysis.

DATA AND DATA REDUCTION

The wind data used in this study was obtained from rocketsonde observations during the period January 1970 to December 1971. The data were reduced from tapes provided by the National Aeronautics and Space Administration (Wallops Island, Virginia), by the U. S. Air Force (Air Weather Service, E.T.A.C., Scott A.F.B., Illinois) and from publications of World Data Center A (Nos. 1-12, Vol. 7, 1970, and Nos. 1-12, Vol. 8, 1971).

The observation locations used in this study are:

1. Ascension Island, A.F.B. (AFETR) (07°59'S, 14° 25'W)
2. Fort Sherman, Canal Zone (09°20'N, 79° 59'W)
3. Kwajalein, U.S.A.M.C., Marshall Islands (08°44'N, 167° 44'E)

Several different rocket types were used at the stations during the period to measure winds in the stratosphere and mesosphere and the accuracy of all the rocketsonde data is considered to be within $\pm$ 3 meters per second.

There were several periods of missing data and the most noticeable are from January 1970 to May 1970 at Fort Sherman and November 1970 to January 1971 at Ascension Island. All stations had some missing data during the Christmas season. Additionally, there are a few cases where no observations occurred for periods of 7 to 10 days. Finally, there are occasionally missing sections in the observations, particularly below 30 km and above 50 km, and very infrequently at irregular intervals between 30 km and 50 km. Even with the data blanks and observation blanks, there are often three observations per week and usually a minimum of two observations per week.

In order to have good continuity of the data, only those launches which occurred within two hours of the average rocket launch time at each station were considered. Next, a five point or four kilometer vertical smoothing was accomplished. If no observations are available within one or two kilometers above or below a level, then that level is eliminated from the data. After the five point smoothing the mean winds at five kilometer levels from 30 km to 55 km were determined.

Since launches are usually on Monday, Wednesday, or Friday, it is clear the unequally spaced observations are normally periodic in some multiple of seven days. For this reason, multiples of seven days are used as the period of analysis for the energy spectrum. The seven-day multiples allowed maximum repetition of correlations for a fixed time. In view of the data blanks and seasonal changes, the primary period of analysis is 84 days, beginning in March 1970. This allowed the following periods to be analyzed:

1. Ascension Island (south of the equator)
   a. fall, winter and spring (1970) (east-west and north-south)
   b. winter and spring (1971) (east-west and north-south)
2. Fort Sherman
   a. summer and fall (1970) (east-west and north-south)
   b. summer and fall (1971) (east-west and north-south)
3. Kwajalein
   a. spring, summer and fall (1970) (east-west and north-south)
   b. spring and summer (1971) (east-west)
   c. summer (1971) (north-south)

In consideration of the computational limitations and the numerical instability of the analysis techniques, a trend is removed and the data smoothed to allow for analysis of 9 to 11 waves. The following two periods are analyzed:

1. An 84 day trend removal and an 8 day smoothing;
2. a 42 day trend removal and a 4 day smoothing.

Both of these periods eliminated from the analysis the tidal waves, many of the gravity waves and most of the high frequency (period less than 4 days) synoptic waves.

To remove the trend and smooth the data, all data points of delta time of 24 hours that were missing were linearly interpolated. The interpolated and real data points were used to remove the previously selected trends from the real data. The same procedure is used to smooth the data over the selected periods. However, after removal of the trend and smoothing of the high frequency waves, the original data points are the only ones used for any analysis.

Next, the average variances and standard deviations of the detrended and smoothed data are determined at 30, 35, 40, 45 and 50 km for ten-day periods and seasonal periods. Finally, the covariances are computed and normalized for all possible times based on 84 and 42 day periods. These normalized covariances or correlation coefficients are then used for the time series analysis.

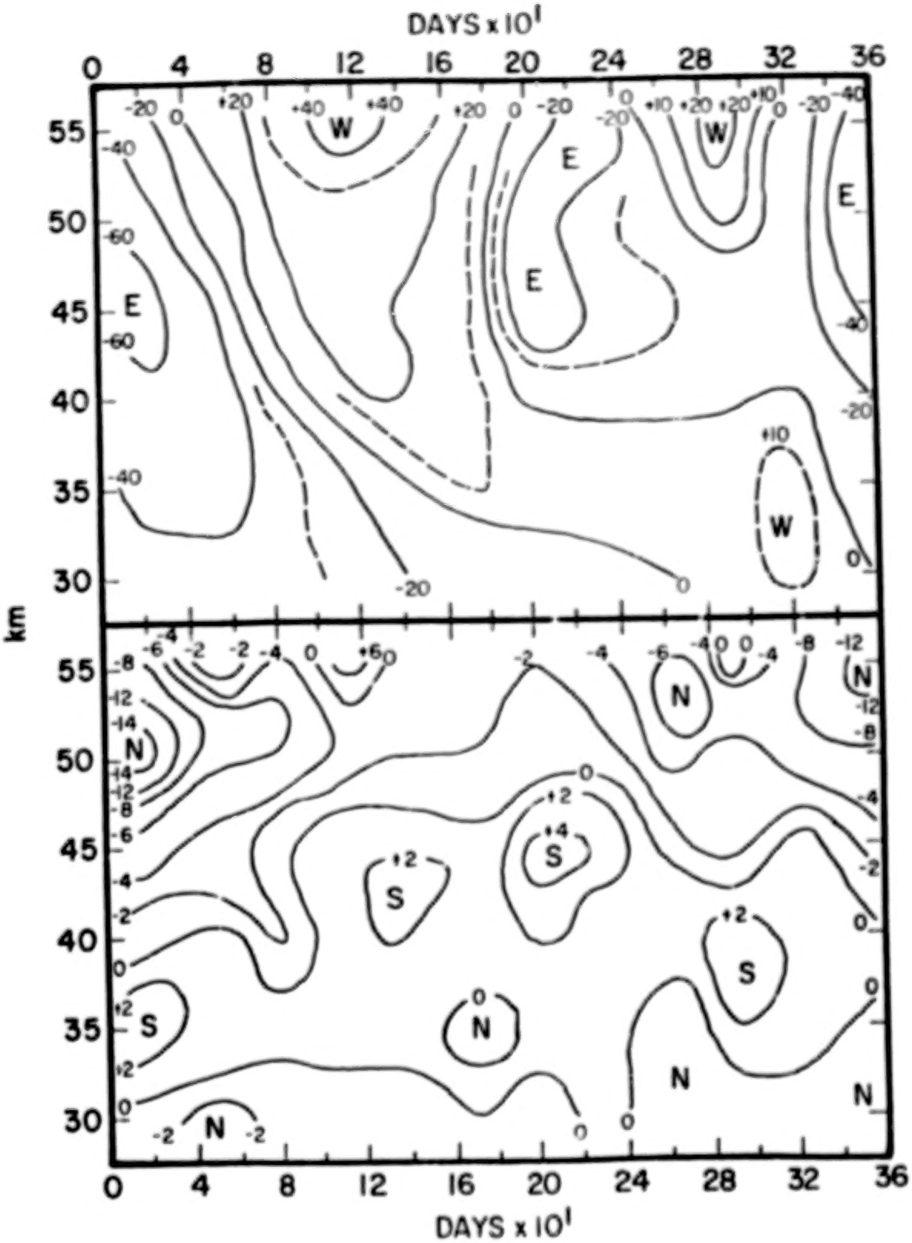
ANALYSIS OF WINDS, AMPLITUDES AND CORRELATIONS

The mean winds are determined at five km levels from 30 km to 50 km after the raw data has undergone a five point vertical smoothing. The smoothed data are divided into thirty-day periods beginning on 1 January. These thirty-day periods are, in turn, divided into three ten-day periods. All observations occurring in a ten-day period are weighted equally and their sum normalized. Each of the three ten-day periods are, in turn, summed and normalized. Figures 1-4 show examples of the resulting averages for 1970 at Ascension Island and Kwajalein.

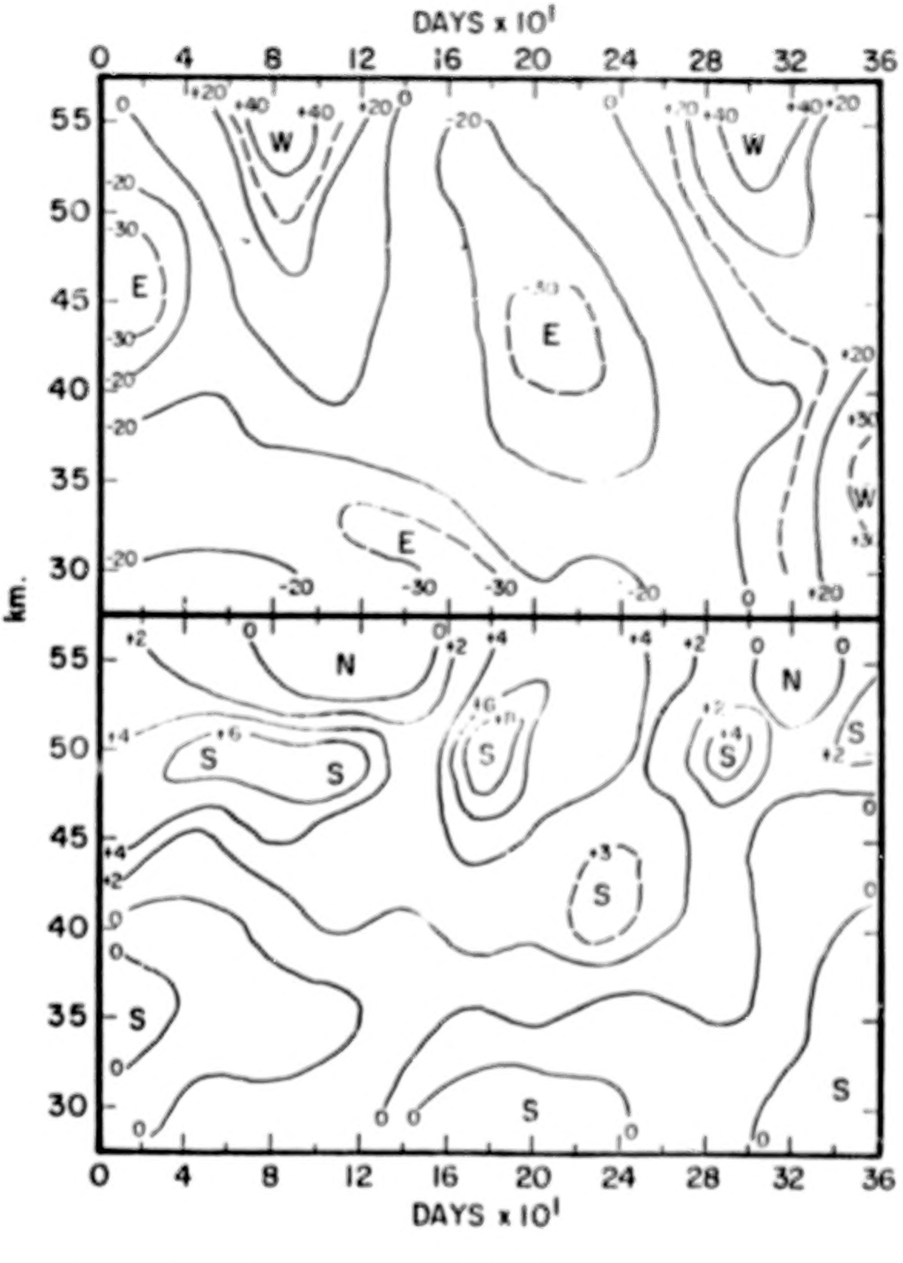
It can be seen that the easterlies and westerlies at Ascension Island and Kwajalein are very similar for all periods. This is true for the east-west component of velocity at all stations and for all periods. However, Ascension Island did show slightly stronger easterlies than Fort Sherman and Kwajalein. The north-south component of velocity did not indicate the flow was as general at all stations, but the late summer southerlies were present at all stations, particularly between 40 km and 55 km. It should be noted that the north-south winds are very weak compared with the east-west winds, particularly below 40 km. Generally speaking, the total kinetic energy appears to increase with height for both components from 30 km to 55 km.

In order to analyze the eddy or turbulent kinetic energy of the winds, the mean or trend must be removed. First, the trend was removed. After the data were detrended and smoothed, an analysis was made of the variances and standard deviations. Table 1 shows an example of the seasonal values. The seasonal means appear to be most significant and the results are shown in Figures 5-10. Figures 5-7 are the north-south and east-west variances at Kwajalein for the spring, summer and fall of 1970 at 30 km to 50 km, and Figures 8-10 are the same for Ascension Island for the fall, winter and spring of 1970.

For all three stations and at most levels and seasons, the north-south velocity component showed more total variance for the 42 day to 4 day period than for the 84 day to 8 day period. This would indicate the synoptic scale waves with periods from 4 days to 8 days have slightly



Figures 1 and 2. Mean winds [m/sec] Ascension Island, 1970, east-west and north-south.



Figures 3 and 4. Mean winds [m/sec] Kwajalein, 1970, east-west and north-south.

Table 1. Mean Seasonal Variance [m^2/sec^2]

Kwajalein, 1970, Period 84 days to 8 days

East-West

Alt. Km	Spring Var.	Spring Std. D.	Summer Var.	Summer Std. D.	Fall Var.	Fall Std. D.
30	6.52	2.55	6.39	2.53	7.05	2.66
35	7.44	2.73	24.19	4.92	15.38	3.92
40	28.27	5.32	41.03	6.41	15.84	3.98
45	166.43	12.90	48.47	6.96	17.47	4.18
50	257.42	16.04	105.11	10.25	97.76	9.89

North-South

Alt. Km	Spring Var.	Spring Std. D.	Summer Var.	Summer Std. D.	Fall Var.	Fall Std. D.
30	2.78	1.67	1.40	1.18	3.48	1.86
35	5.51	2.35	4.17	2.04	0.89	0.94
40	5.54	2.35	4.03	2.01	2.42	1.56
45	7.99	2.83	4.54	2.13	5.47	2.34
50	9.08	3.01	8.86	2.98	13.44	3.67

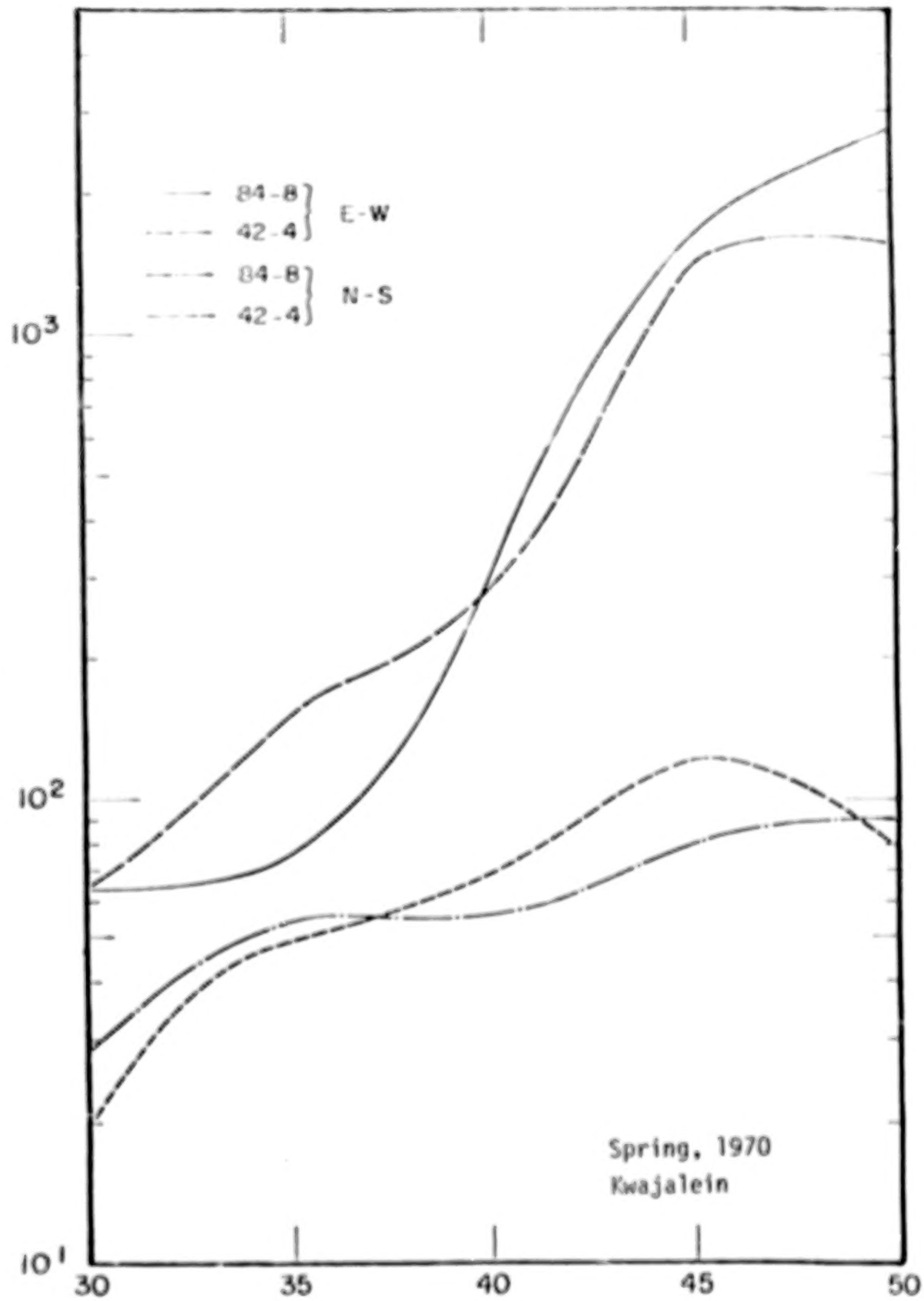


Figure 5. Mean Variance

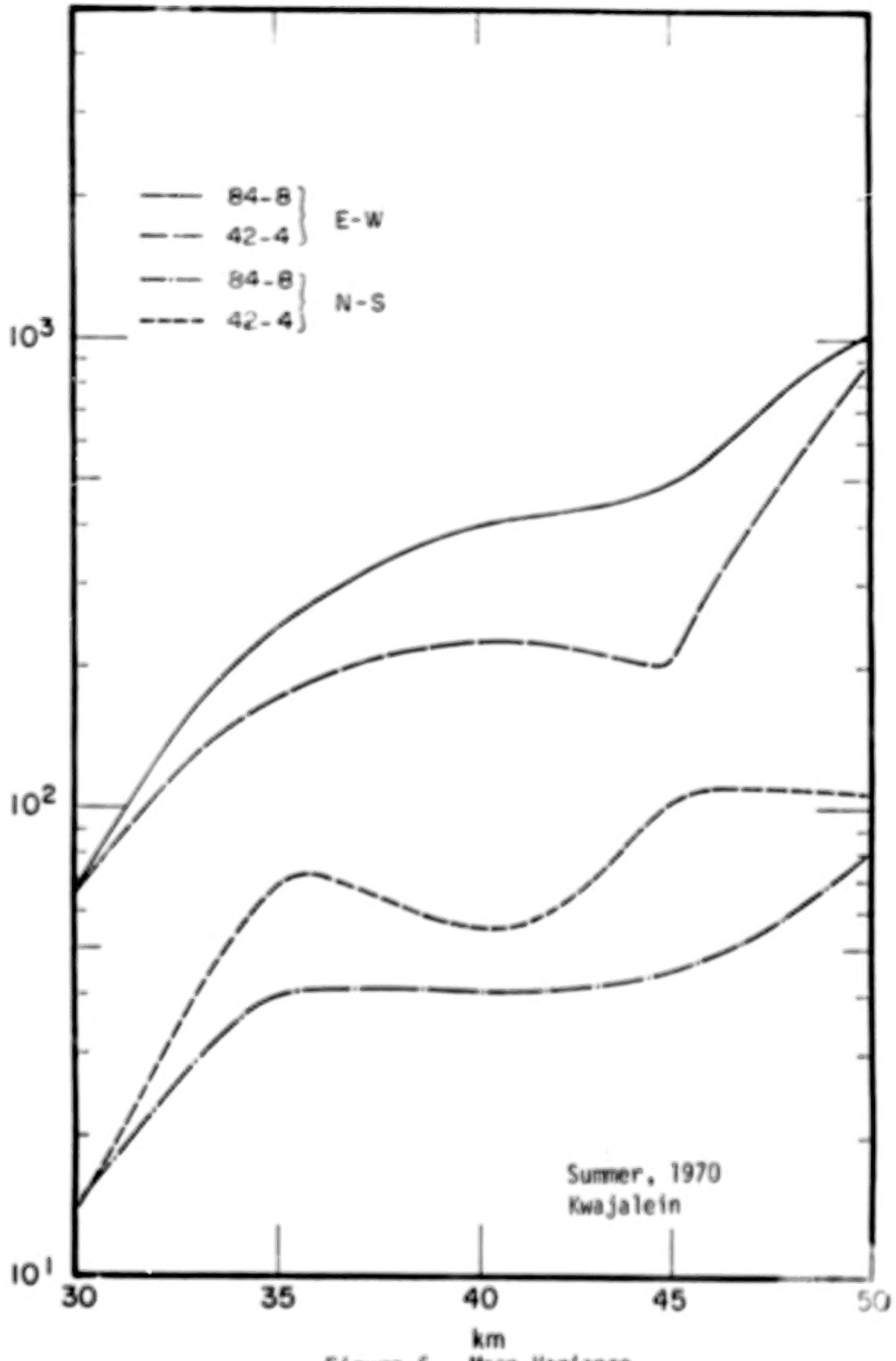


Figure 6. Mean Variance

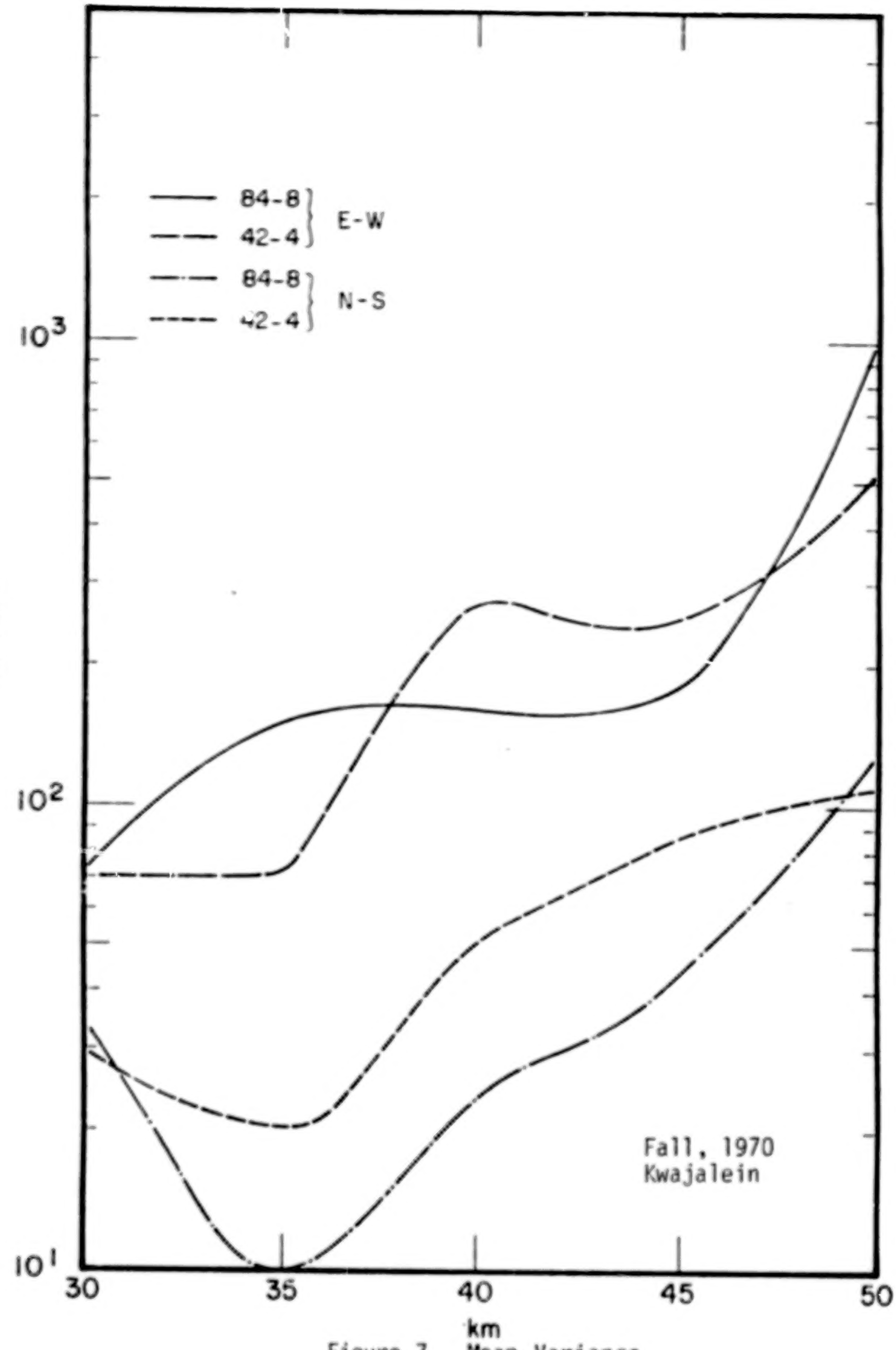


Figure 7. Mean Variance

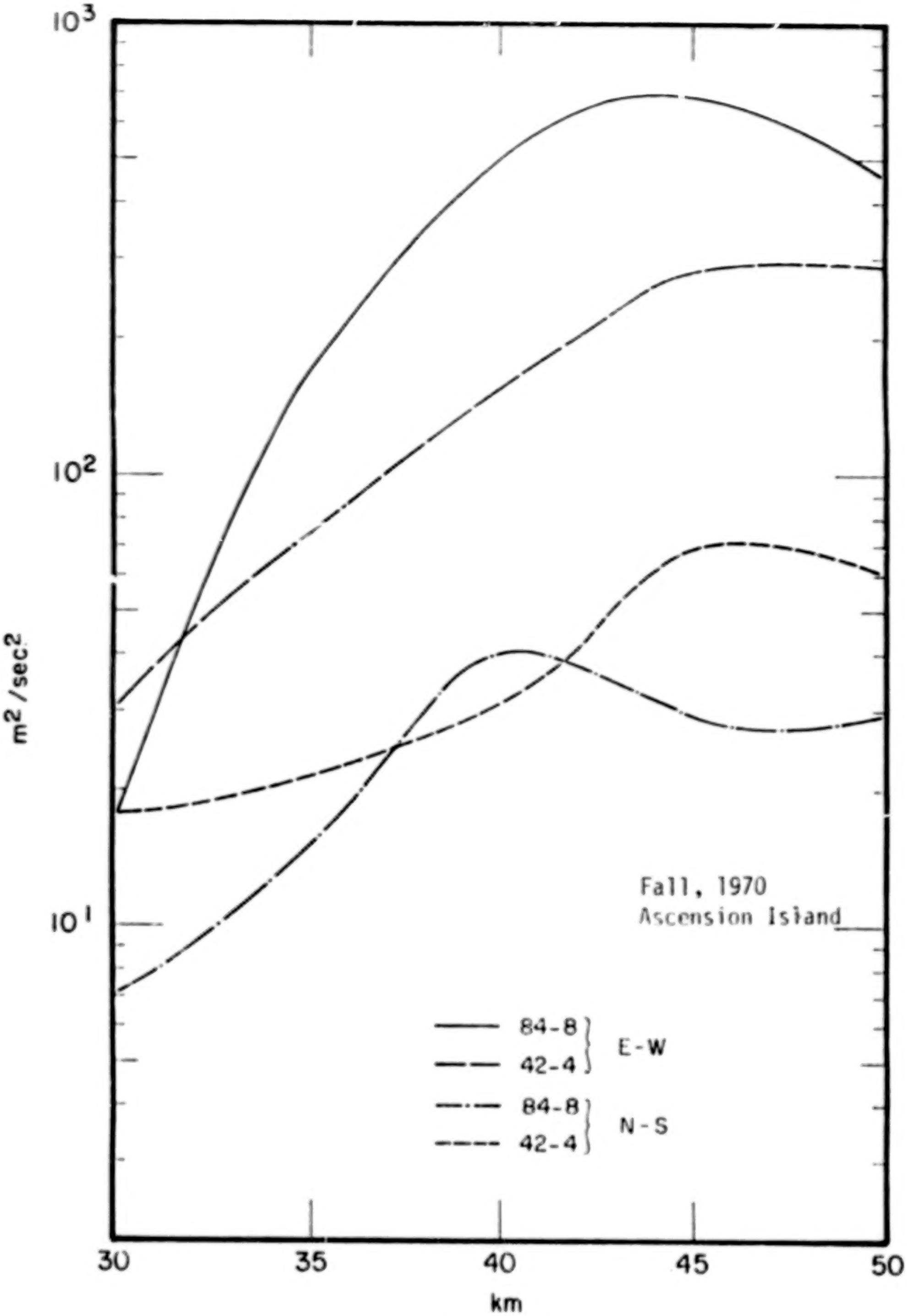


Figure 8. Mean Variance

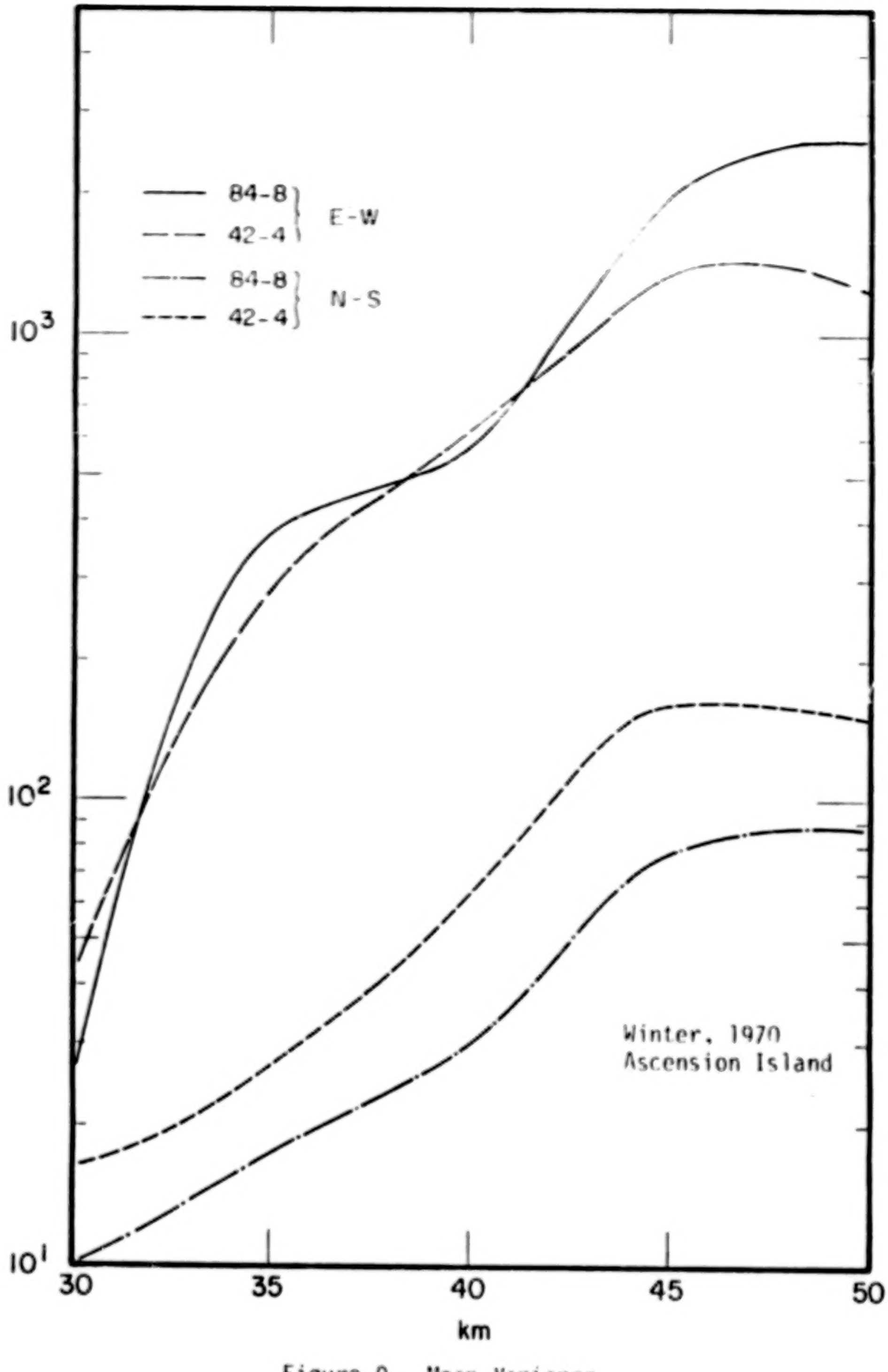


Figure 9. Mean Variance

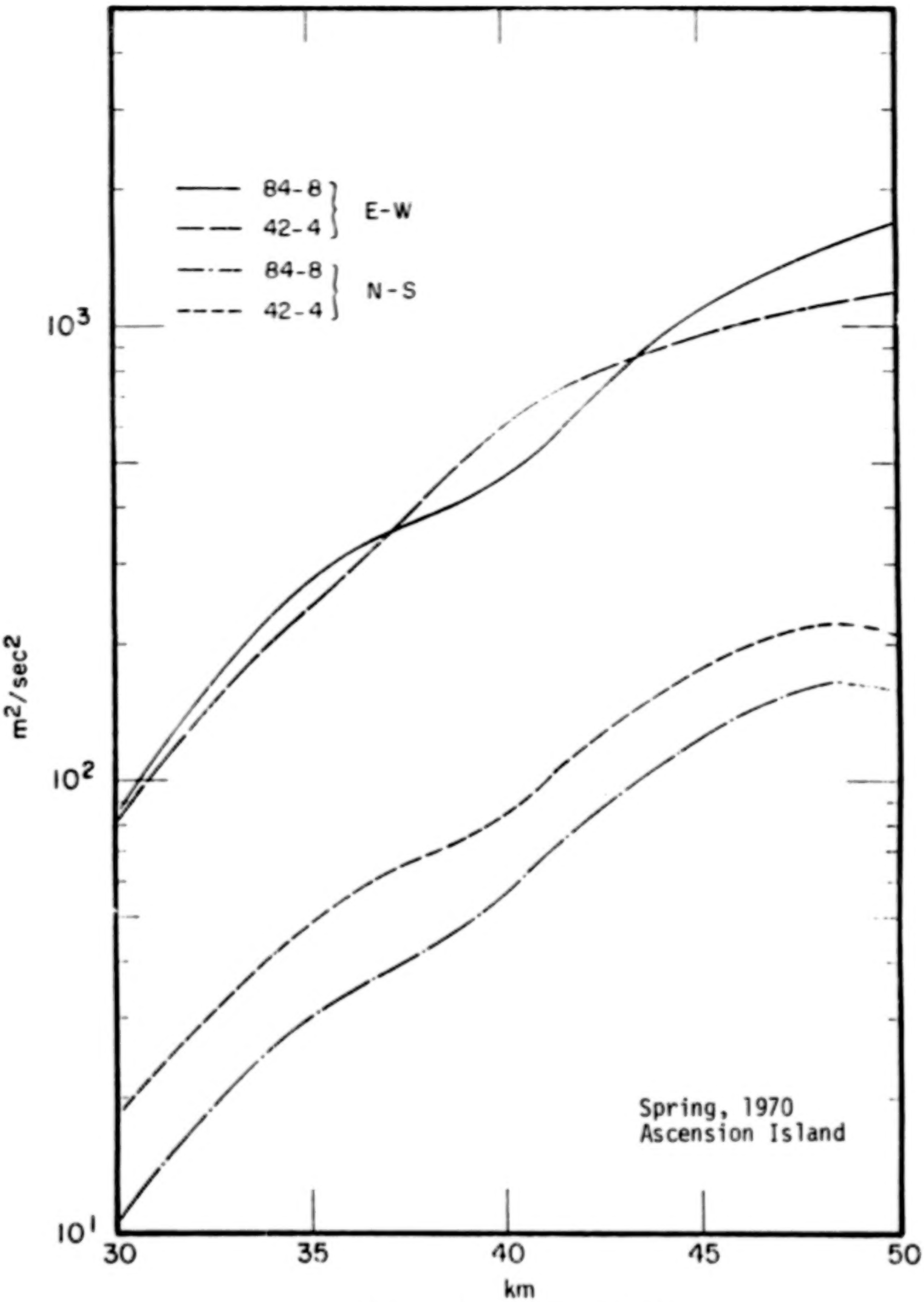


Figure 10. Mean Variance

greater energy than the seasonal waves with periods from 42 days to 84 days. The results of the spectral analysis of the north-south winds indicate a significant energy peak in the synoptic scale waves. In the case of the east-west winds the reverse is true, particularly with respect to the strong easterlies. While there is a known direct relationship between both the magnitude of the mean winds and the eddy kinetic energy, and the variance and the eddy kinetic energy, analysis of the mean wind shows the vertical increase in the total variance is more directly related to the vertical changes in the relative magnitude of the winds with respect to one another and not just the wind speed.

After the data is detrended and smoothed, covariances are computed for each pair of observations based on the period of detrending and delta time. The covariances are then summed and normalized for each season with respect to the total number of pairs with the same delta time. Table 2 contains a sample for the north-south velocity component at Fort Sherman during the summer of 1970 at 50 km.

As can be seen from Table 2, there are some questionable normalized covariances (those underlined in the table), particularly those determined using few pairs of covariances. Therefore, the covariances are evaluated for obviously biased results (such as a covariance larger than the mean seasonal variance). Samples of the resulting correlations are plotted in Figures 11-14. Figures 11 and 12 show the correlation of the east-west and north-south components at Fort Sherman in the summer of 1970 for the 42 day to 4 day period. There is a more consistent pattern for the north-south component, and this pattern is present at all stations and all seasons. Figures 13 and 14 are plots of the north-south component for the period 84 days to 8 days at Kwajalein and Ascension Island at 45 km in 1970. These figures indicate better correlation (more consistent pattern) at Ascension Island. For both cases above, the more consistent correlation (oscillation toward the zero) also leads to more stable solutions of the energy spectrum.

Table 2. Normalized Covariance

Fort Sherman, Summer, 1970, 50 km

ΔT (days)	Period 84-8		Period 42-4	
	No. of Pairs	Cor. Coef.	No. of Pairs	Cor. Coef.
1	2	8.90	3	2.97
2	14	107.38	19	17.88
3	8	96.53	11	2.69
4	8	83.16	11	7.35
5	13	73.98	19	- 0.04
6	4	8.70	5	- 1.51
7	18	53.95	27	2.37
8	4	9.20	5	0.00
9	12	17.00	18	4.69
10	8	21.66	11	3.78
11	6	22.25	11	2.37
12	12	1.41	18	1.42
13	3	4.07	5	- 5.36
14	16	15.58	27	- 1.84
15	2	- 3.41	5	- 6.22
16	11	24.94	19	- 3.79
17	6	9.98	11	- 5.39
18	6	56.73	11	- 2.49
19	10	29.01	18	- 6.92
20	2	- 31.70	4	1.61
21	14	55.32	15	- 6.67
22	2	- 32.33		
23	9	62.04		
24	5	17.45		
25	5	60.76		
26	8	27.71		
27	2	- 21.04		
28	12	14.09		
29	2	- 13.49		
30	8	- 8.12		
31	5	- 15.56		
32	5	- 15.48		
33	8	- 32.11		
34	1	- 2.08		
35	13	- 40.49		
36	2	- 7.09		
37	8	- 51.48		
38	4	- 41.85		
39	6	- 41.12		
40	7	- 48.41		
41	4	- 36.99		
42	5	- 56.86		

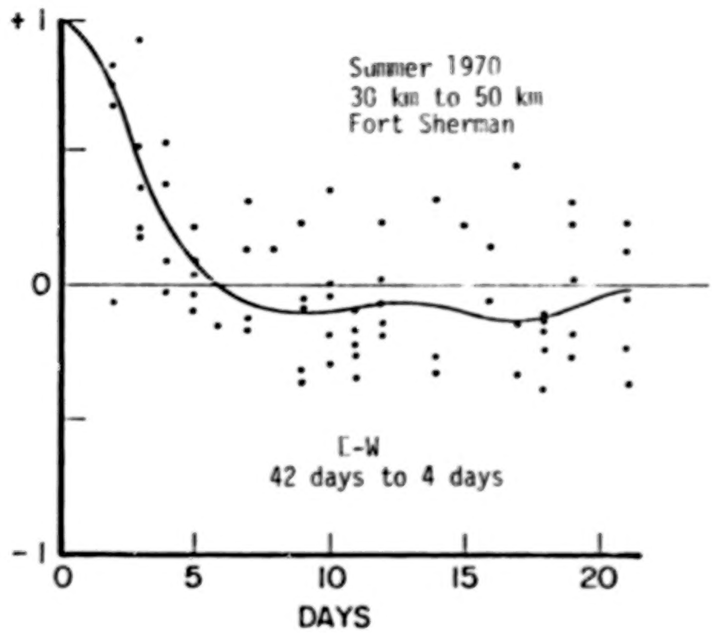


Figure 11. Normalized correlation coefficients.

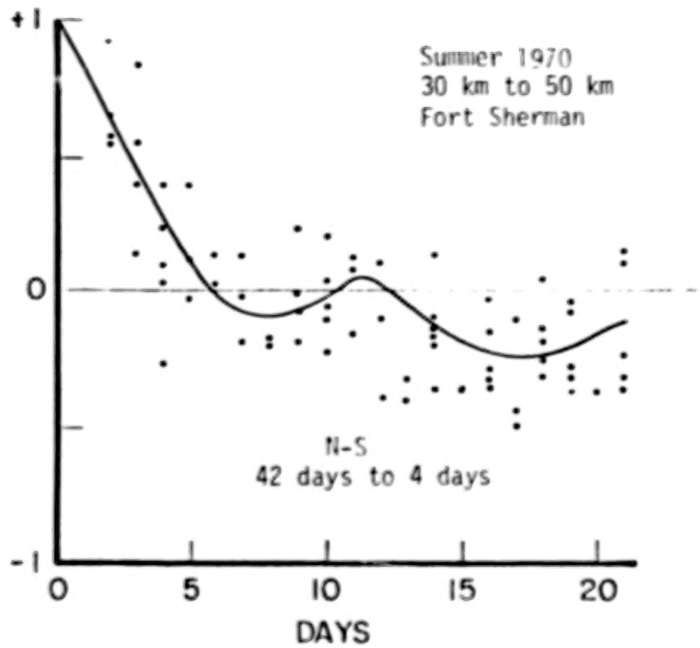
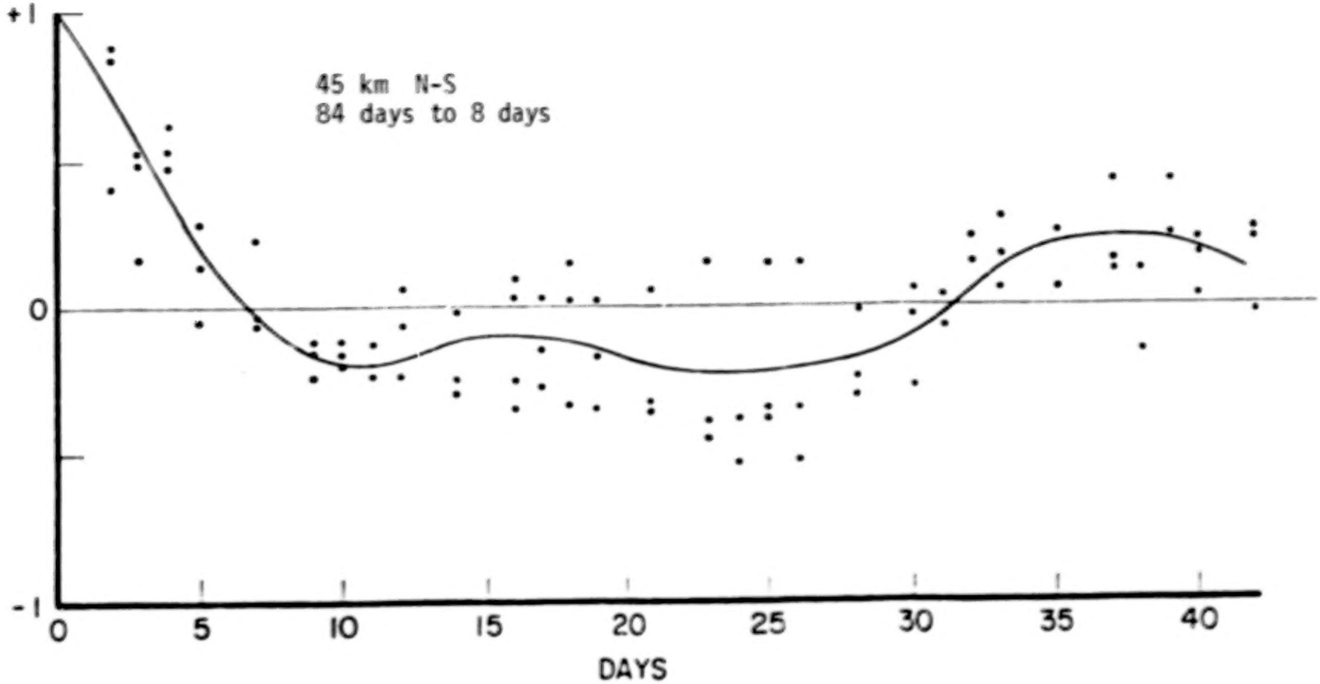
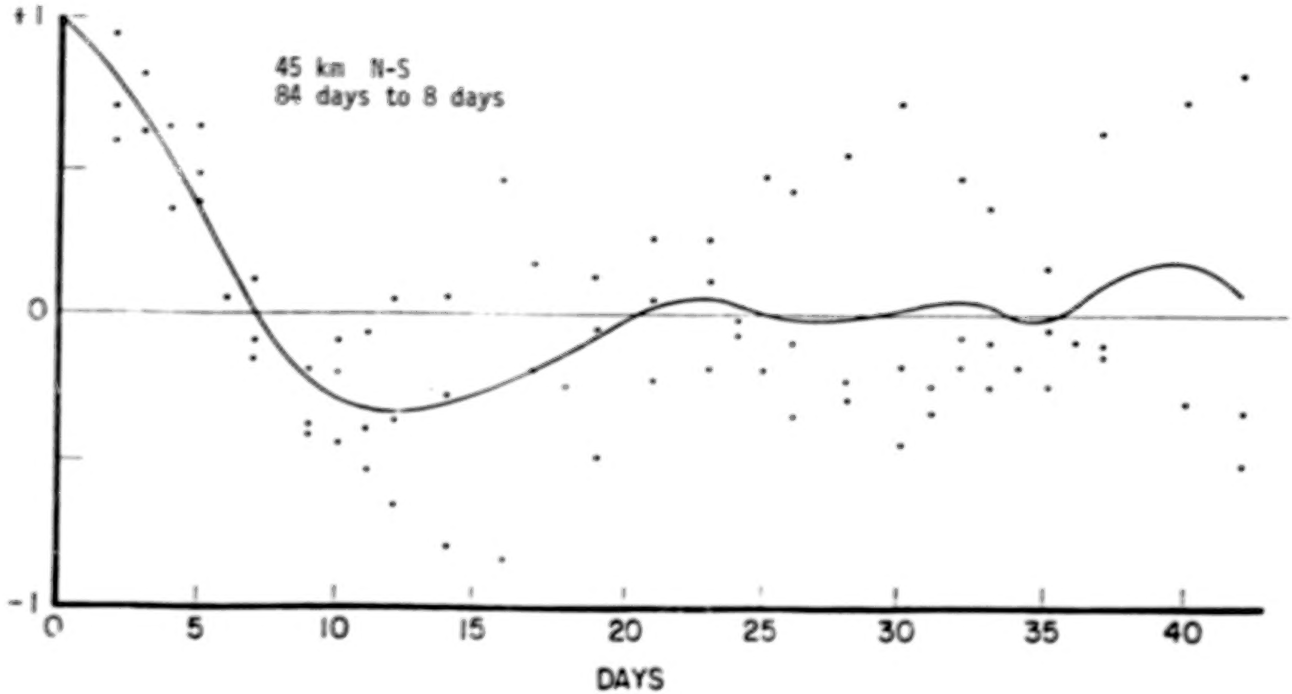


Figure 12. Normalized correlation coefficients.



Kwajalein, 1970

Figure 13. Normalized correlation coefficients



Ascension Island

Figure 14. Normalized correlation coefficients

THE EULERIAN TIME SPECTRA

As noted at the beginning of this article, most work in the stratosphere and mesosphere has used extensive interpolation. All analyses of the Eulerian time spectra use some type of trend removal, smoothing and interpolation to an equally spaced grid, and then harmonics of Fourier analysis of the equally spaced data is accomplished. Since frequently nearly half of the data points used are interpolated by some analysis scheme, it is obvious that the scheme could and probably does have a considerable influence or bias on the end result. Therefore, in this study the analysis of the Eulerian time spectra is accomplished using only the actual observations. Due to computational problems, it is necessary to remove an approximate trend and accomplish some very simple smoothing.

It can be shown if a process is statistically stationary and circular, then the process has an expected value function which is dependent on the interval between two observations on the unit circle. This function is of the form:

$$\sum_{k=-\infty}^{\infty} Z_k \exp \{i\ 2\pi\ k\ D_{i,j}\} = E[x(\lambda_i)\ \overline{x(\lambda_j)}] ,$$

and for time series analysis the relationship becomes:

$$\sum_{n=-\infty}^{\infty} S_n \exp \{i\ 2\pi\ n\ T_{i,j}\} = E[x(t_i)\ \overline{x(t_j)}] .$$

Since this process is circular and only a function of the distance between two points on the unit circle, and if we assume the process to be real and:

$$T_{i,j} = [(\ |t_i - t_j|)/T]$$

where T is the total time period, then it follows that:

$$0 < T_{i,j} \leq .5 .$$

Finally, it can be shown that the expected value function actually reduces to a covariance or correlation function (for zero mean) and in the case of a real process, such as the north-south or east-west component of velocity, it follows that:

$$E[u(t_i)\ \overline{u(t_j)}] = \langle u(t)\ u(t + T_{i,j})\rangle \quad ,$$

where

$$\langle\quad\rangle = \frac{1}{N}\sum_{\ell=1}^{N} (u(t_\ell)\ u(t_\ell + T_{i,j})) \quad .$$

Since u(t) is assumed to be a random process and any random process has a spectral representation similar to:

$$u(\Delta t) = \sum_{n=-\infty}^{\infty} (\alpha_n + i\ \beta_n)\ \exp\ (i\ 2\pi\ n\ \Delta T) \quad ,$$

then $S_n = E|\alpha_n + i\ \beta_n|^2$.

Further,

$$\sum_{n=-\infty}^{\infty} S_n \exp\ (2\pi\ in\ \Delta t) = \sum_{n=-\infty}^{\infty} S_n[\cos(2\pi\ n\ \Delta T) + i\sin(2\pi\ n\ \Delta T)] \ .$$

Now let

$$R\ (T_{i,j}) = \langle u(t)\ u(t + T_{i,j})\rangle \quad .$$

Since the process is real and we are only interested in the real solutions, the problem reduces to solving:

$$\sum_{n=-\infty}^{\infty} S_n\ [\cos(2\pi\ n\ T_{i,j})] = R(T_{i,j})$$

and this, in turn, becomes:

$$S_o + 2 \sum_{n=1}^{\infty} S_n[\cos(2\pi\ n\ T_{i,j})] = R(T_{i,j}) .$$

If the function has a zero mean, then $u(t)$ represents only the perturbations from the mean of the process, and $R(T_{i,j})$ is the autocorrelation function.

Now let $\Delta t_k = T_{i,j}$ and return to the original equation. Depending on the mean of the function $R(\Delta t_k)$, two forms of the problem are possible. They are:

$$\begin{bmatrix} 1 & 2[\cos(2\pi\ \Delta t_1)] & . . . & 2[\cos(2\pi\ \Delta T_1)(n-1))] \\ 1 & . & & \\ & . & & \\ & . & & \\ 1 & 2[\cos(2\pi\ \Delta t_n)] & . . . & 2[\cos(2\pi\ \Delta T_n(n-1))] \end{bmatrix} \bullet \begin{bmatrix} S_o \\ . \\ . \\ . \\ S_{n-1} \end{bmatrix} = \begin{bmatrix} R(\Delta t_1) \\ . \\ . \\ . \\ R(\Delta T_n) \end{bmatrix}$$

or

$$2 \begin{bmatrix} \cos(2\pi\ \Delta T_1) & . . . & \cos(2\pi\ \Delta t_1(n)) \\ . & & . \\ . & & . \\ . & & . \\ \cos(2\pi\ \Delta t_n) & . . . & \cos(2\pi\ \Delta T_n(n)) \end{bmatrix} \bullet \begin{bmatrix} S_1 \\ . \\ . \\ . \\ S_n \end{bmatrix} = \begin{bmatrix} R(\Delta t_1) \\ . \\ . \\ . \\ R(\Delta t_n) \end{bmatrix}$$

The problem is now reduced for both equations above to solving a system of linear equations.

The work of Julian and Kline (1974) is one of the first in analyzing the energy spectrum of unequally spaced meteorological data. The techniques used are described in Hanson and Lawson (1974) and the program used in this study also is obtained from Hanson and Lawson (1974). The technique is solving the constrained least-squares of:

$$||A\hat{x} - \hat{b}|| \quad \text{minimized} ,$$

such that $\hat{b} \geq 0$.

One of the constraints of the least-squares method is that the matrix generated by Δt_k is positive definite. Otherwise, the program may not iterate toward an actual solution.

Initially several small matrices based on the first few Δt_k's were tested and all were positive definite. However, it was noted that even when a matrix was intentionally generated that was not positive definite, the general shape of the solution vector was still within the error limits of the program. It appears that only for very large matrices is there a problem with a matrix not being positive definite. In fact, no matrices were generated throughout this study that could not be iterated to an approximate solution by the least-squares method.

It did appear from analysis of the solution vector of some of the larger dimensioned matrices (30 x 30) that the results would have an oscillation about the approximate solution and falsely aliase some energy into the lower frequency waves and in other cases into the higher frequency waves. This was most obvious when no zero wave was included. But, even under these conditions, the relative maximums and minimums were consistent.

The final analysis of the Eulerian time spectra presented in this study was conducted on two banded frequencies by season and at 5 km intervals from 30 km to 50 km. Some of the results are shown in Figures 15 to 21, while the general features will be discussed below. In these figures the normalized energy spectra are the spectra divided by the variance of the velocity.

The east-west components of velocity for the frequency band with period 84 days to 8 days and at 10 km levels from 30 km to 50 km are shown in Figures 15 to 17. The same levels for the north-south component are shown in Figures 18 to 20.

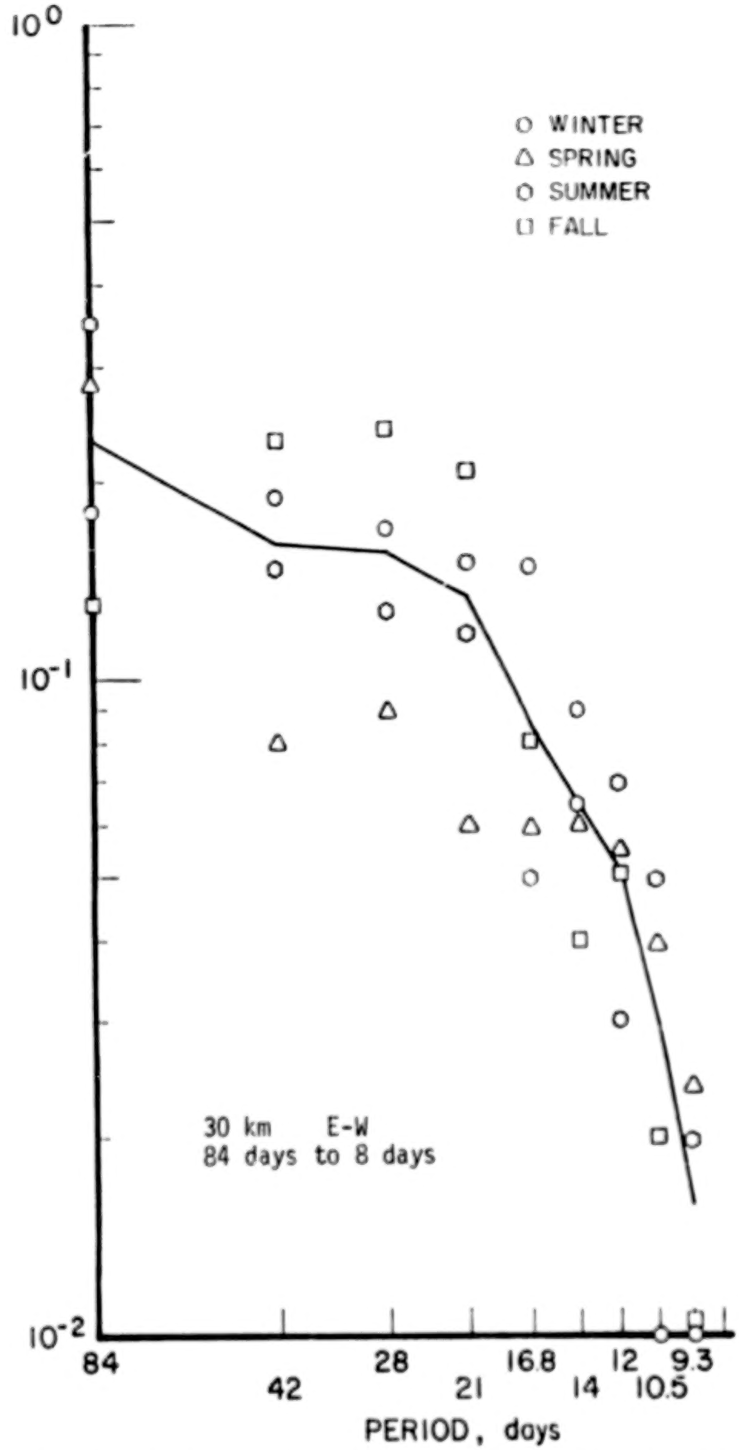


Figure 15. Normalized energy spectrum for east-west wind.

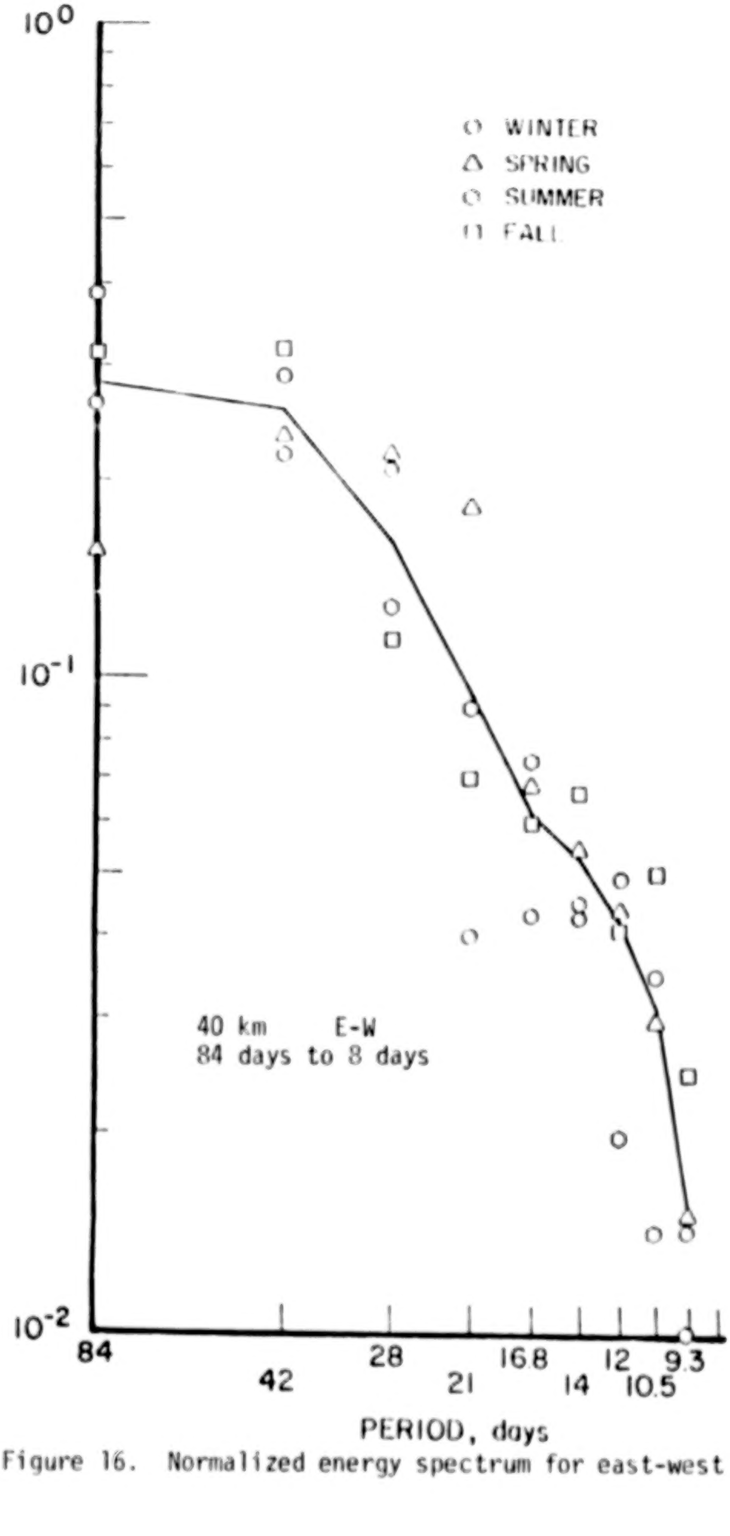


Figure 16. Normalized energy spectrum for east-west wind.

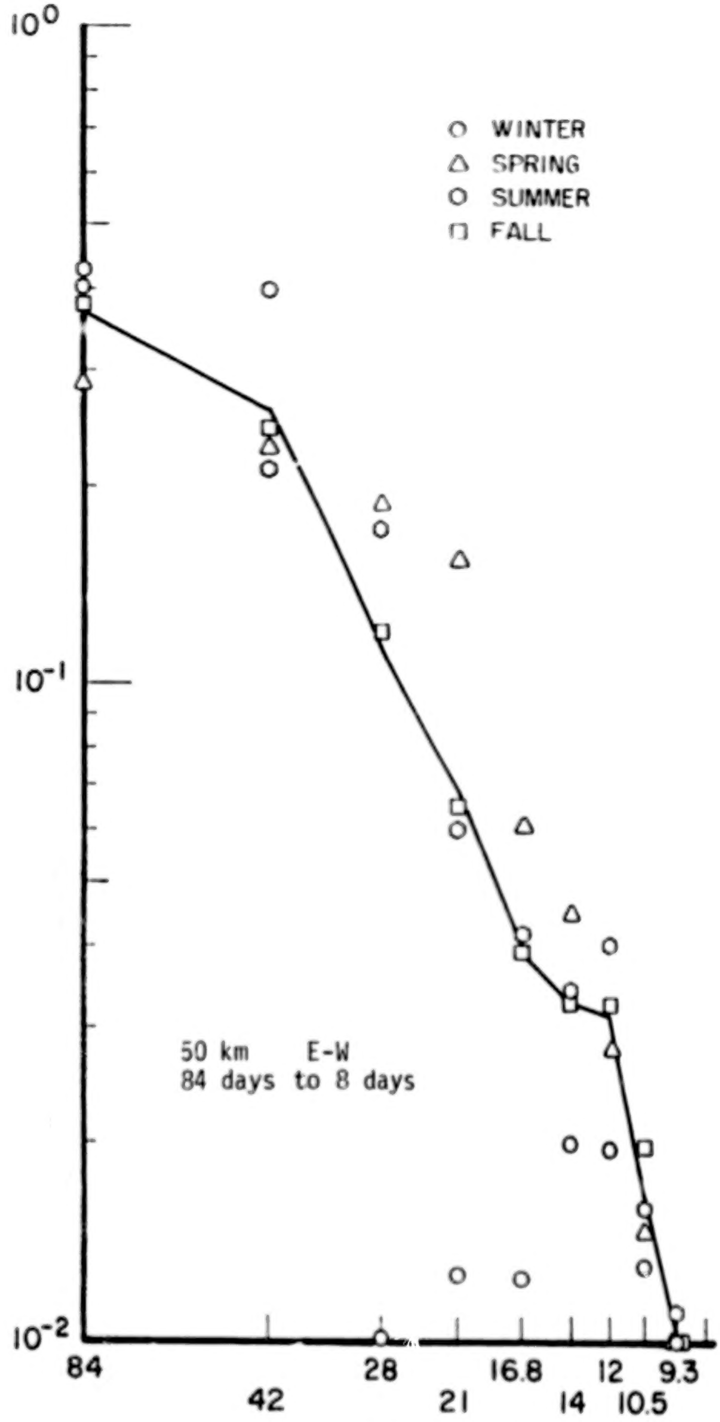


Figure 17. Normalized energy spectrum for east-west wind.

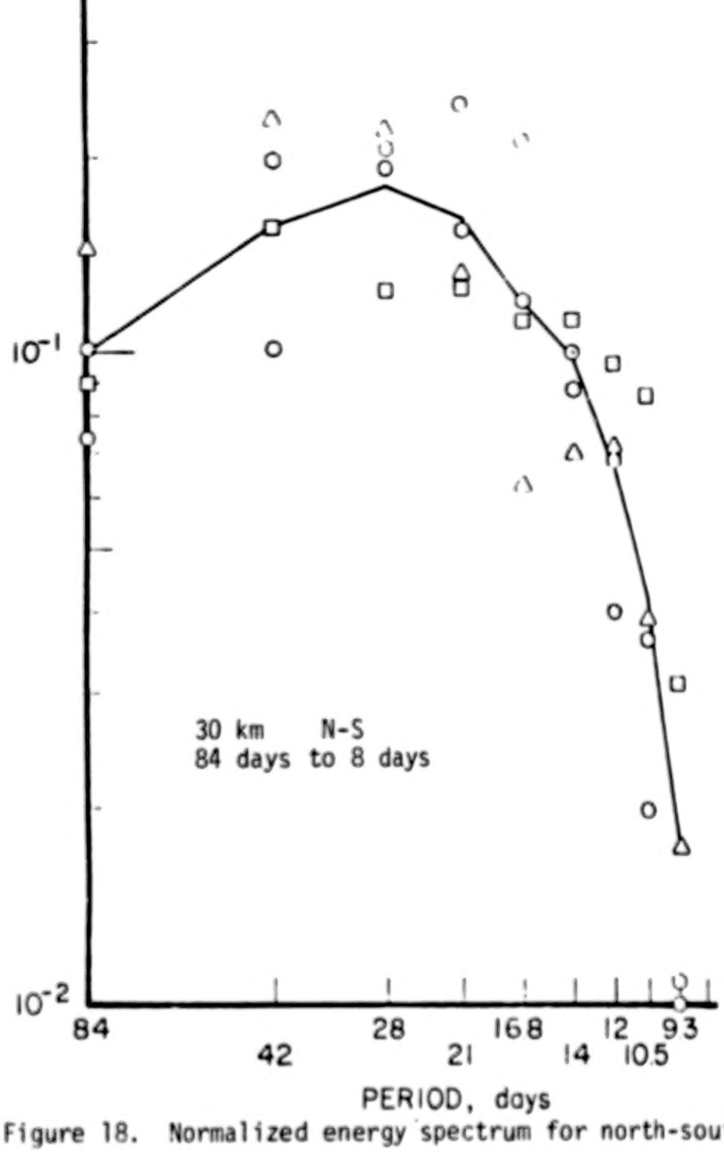


Figure 18. Normalized energy spectrum for north-south wind.

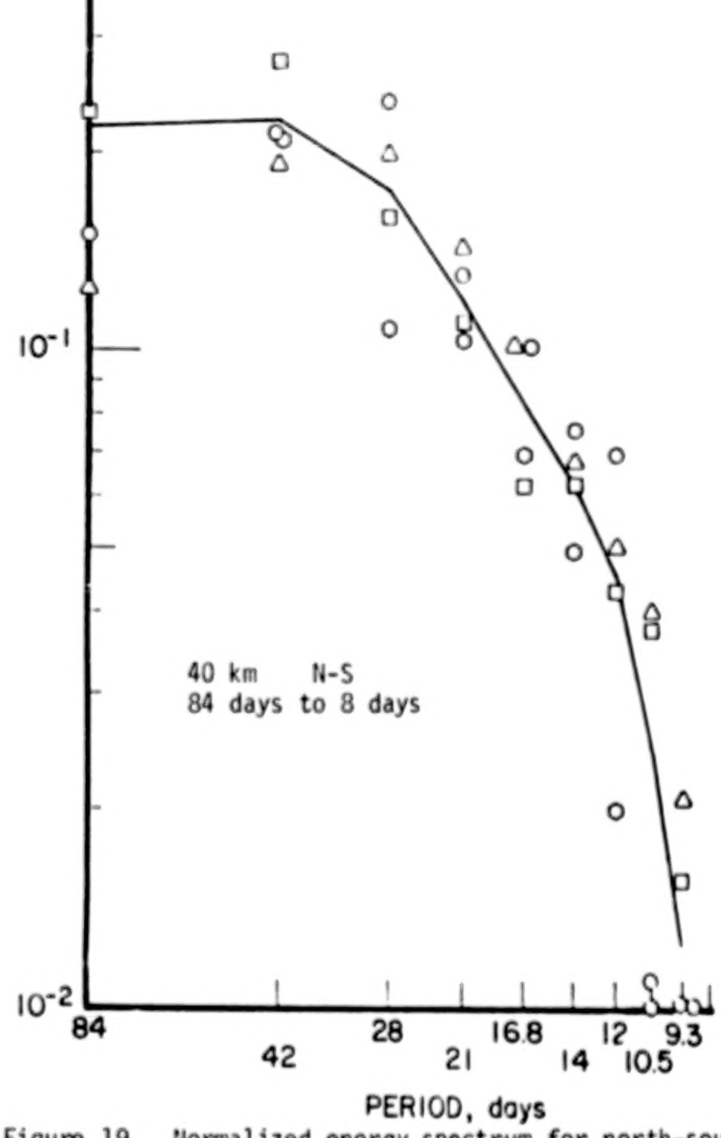


Figure 19. Normalized energy spectrum for north-south wind.

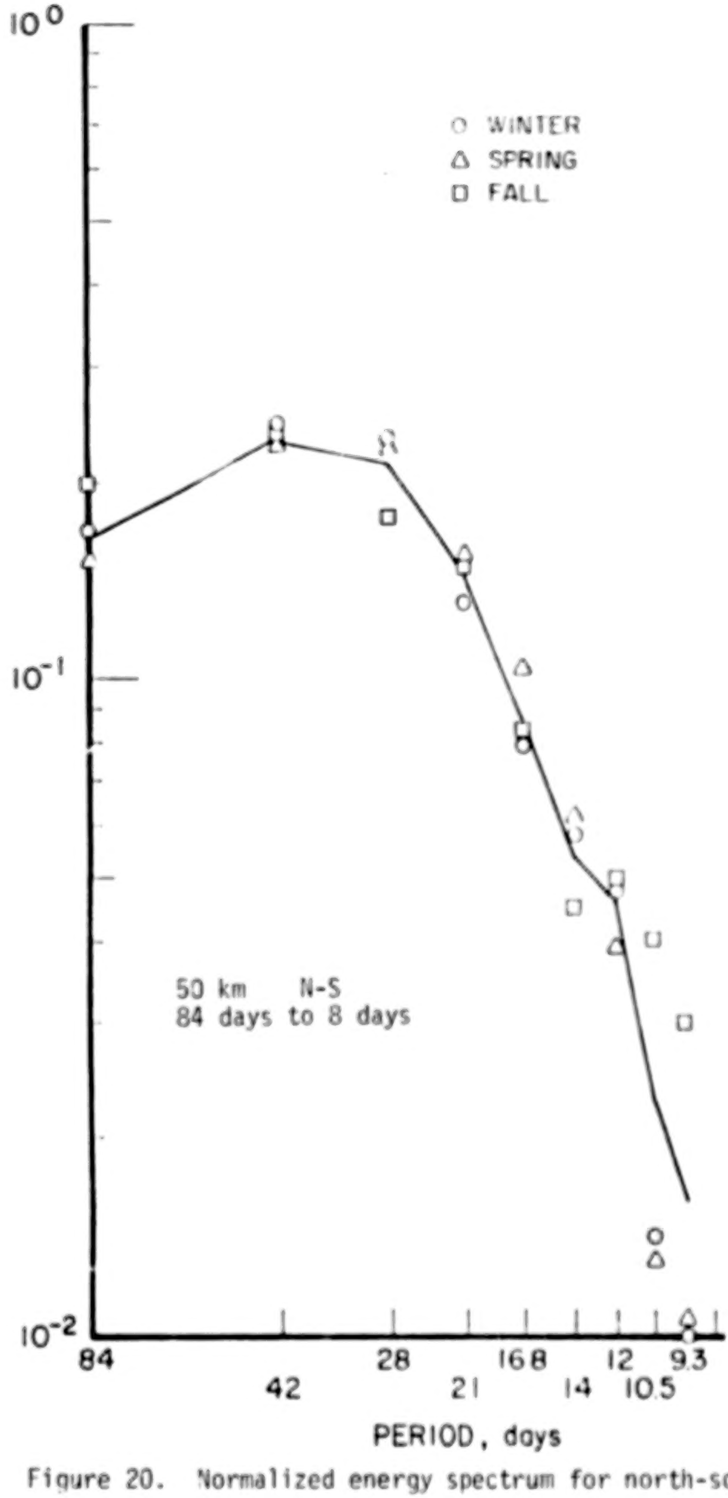


Figure 20. Normalized energy spectrum for north-south wind.

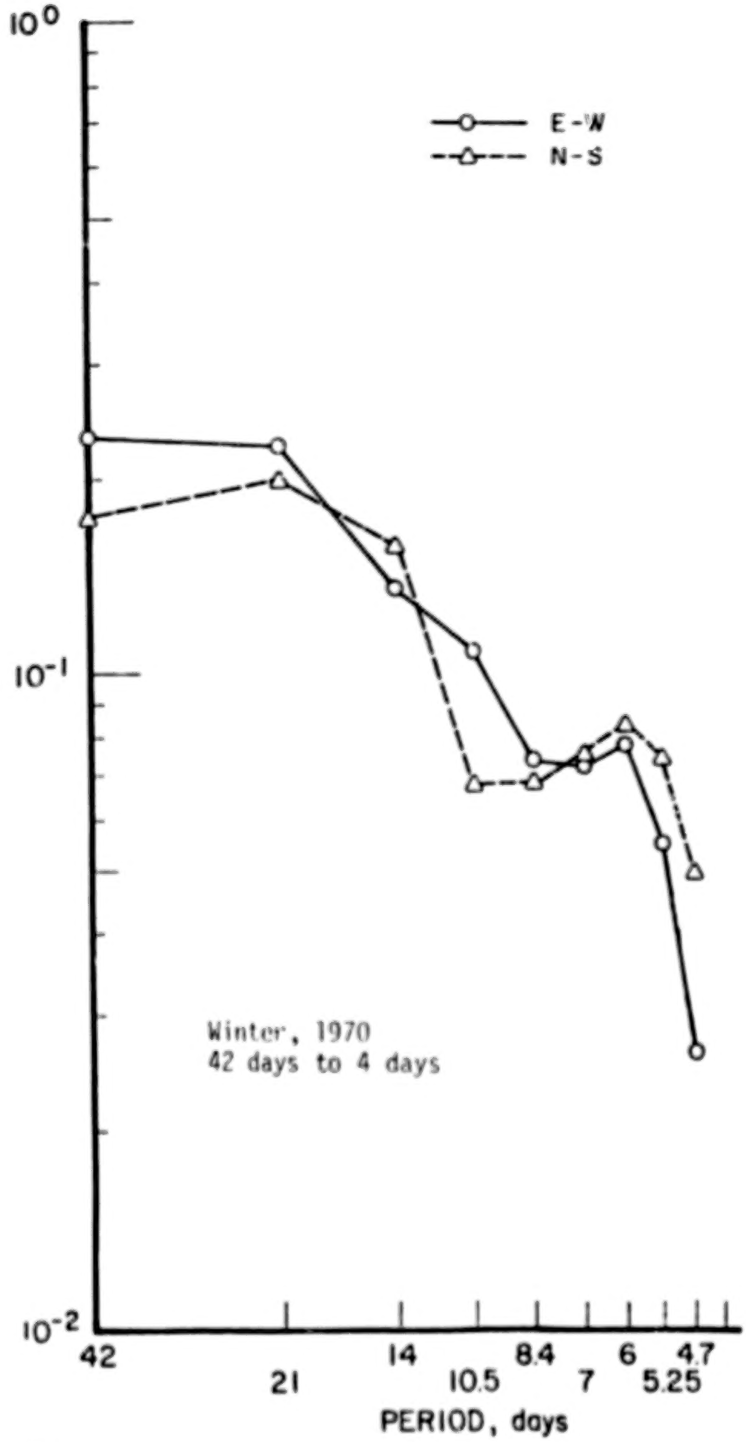


Figure 21. Average normalized energy spectrum from 30 km to 50 km at Ascension Island.

The results show the maximum energy of the east-west velocity component is in the seasonal waves with periods greater than 42 days. This feature is more obvious as the altitude increases from 30 km to 50 km, particularly in the summer at all levels. This feature compares well with Norton (1967), although the present study indicates more spectral energy in the mid-frequency waves.

The north-south velocity component for the same levels and period (except summer at 50 km) shows the spectral peak to be in the mid-frequency (seasonal or monthly) waves with period from 28 days to 42 days.

When the frequency band is reduced to a period of 42 days to 4 days, the general spectral features were changed slightly. Both velocity components showed a synoptic scale wave, and this is seen in Figure 21. However, both components still indicate spectral maximums are in the longer period waves. Analysis of all the results indicates the synoptic scale waves have greatest impact on the north-south spectral component.

The magnitude of the north-south spectral component due to the synoptic scale waves was most significant below 45 km and occurred at all stations and all seasons, particularly in the summer. This effect on the north-south component in the stratosphere is similar to the results of Burpee (1972) in the troposphere over the African continent.

It was found that the detrending and smoothing techniques used for this study were only approximate; and it was, therefore, necessary to solve the system of equations that contained the zero wave. Further, even though there should have been little energy in the high frequency waves of periods less than 4 days or 8 days, when the analysis was reduced to only those waves that should exist, there was sufficient energy left over such that the solution vector was numerically unstable. As an example, for the 42 day to 4 day period, it was necessary to solve a system of equations that contained 12 to 15 waves in order to get a numerically stable solution.

TWO-DIMENSIONAL ANALYSIS

Some of the earliest work in the wavenumber-frequency analysis of meteorological parameters was done by Kao (1968), Kao and Sagendorf (1970), and Kao and Kuczek (1973). The techniques used did, however, require interpolation of the data to an equally spaced grid and then application of detrending and smoothing, and finally, use of the fast Fourier transform to get a solution.

This technique is actually a form of the problem as stated in the previous section, only now the expected value function becomes two-dimensional, as shown below:

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{k,n} \exp\{i2\pi(k\, D_{i,j} + n\, T_{k,\ell}\} = E[x(t_\ell, \lambda_i)\, \overline{x(t_m, \lambda_j})] \ .$$

It is obvious that determination of the right-hand side of the expression is the first major problem. If the process is assumed to be real and have a zero mean, and we are interested in only the real portions of the solution, then the problem becomes simplified as follows:

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{k,n} \exp\{i2\pi(k\, D_{i,j} + n\, T_{\ell,m})\} = \langle x'(\lambda,t)\; x'(\lambda+D_{i,j}, t+T_{\ell,m})\rangle$$

where $\langle\ \rangle$ represents the autocorrelation function in two-dimensions. Under these conditions a solution is theoretically possible, but this actually implies, when considering the wavenumber analysis, that all observations were made at the same time, or the function below exists:

$$\langle x'(\lambda),\ x'(\lambda + \Delta\lambda)\rangle = R(\lambda,\ t{=}0) \ .$$

This condition may not imply equally spaced time observations but it does imply some consistent pattern that has some periodicity starting at the same time at all stations. Frequently, the periodicity will allow for easy interpolation to an equally spaced grid, and this, of course, reduces the problem to only one dimension of unequally spaced data.

Returning to the original problem and assuming there is sufficient data to determine the right-hand side of the equation, then the following equation holds:

$$\sum_{n=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} S_{k,n} \cos(2\pi(k\, D_{i,j} + n\, T_{\ell,m}))$$

$$+ \, i \left[\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{k,n} \mathrm{Sin}(2\pi(k\, D_{i,j} + n\, T_{\ell,m}) \right]$$

$$= \langle x'(\lambda,t)\; x'(\lambda + D_{i,j},\, t + T_{\ell,m}) \rangle \, .$$

Now just consider the real portion of the equation, even though we may expect influence from the imaginary portion, and let $D_{i,j} = \delta_\ell$ and $T_{\ell,m} = t_i$, the system then expands to:

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{k,n} \cos(2\pi(k\delta_\ell + nt_i)) = (\langle x'(\lambda,t)\; x'(\lambda + \delta_\ell,\, t + t_i) \rangle)_R$$

where $(\;)_R$ represents the real portion of the autocorrelation function. Next, let $S_{o,o} \to 0$.

Now the equation can be expressed as:

$$2 \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} S_{k,n} \cos(2\pi(k\delta_\ell + nt_i)) + 2 \sum_{k=1}^{\infty} \sum_{n=-\infty}^{-1} S_{k,n} \cos(2\pi(k\delta_\ell + nt_i)) +$$

$$2 \sum_{k=i}^{\infty} S_{k,o} \cos(2\pi\, k\delta_i) + 2 \sum_{n=1}^{\infty} S_{o,n} \cos(2\pi\, nt_i)$$

$$= (\langle x'(\lambda,t)\; x'(\lambda + \delta_\ell,\, t + t_i) \rangle)_R \, .$$

Expanding over wavenumber forms a matrix of the left-hand side and a vector of the right-hand side.

Realizing each element within the matrix generates a matrix and each element of the solution vector generates a vector, this system is most impractical to attempt to solve with known iterative techniques, and an exact solution would be even more untenable.

In view of the numerical problems, an alternative approximation method is suggested.

If it is assumed that each location, which is unequally spaced on the unit circle, has a reasonably large data base available, and that the data has some consistent periodicity at each location (such as rocketsonde stations where observations are generally periodic in some multiple of 24 hours), then there would appear to be a reasonable solution method.

First, recall that the analysis of the equally spaced data is actually a solution of the autocorrelation function:

$$R(T,\lambda) = \langle x'(T,\lambda)\, x'(T + t,\, \lambda + \delta)\rangle$$

Now consider observation stations where the autocorrelation function exists for $\Delta\lambda = 0$, such as $R(T, \Delta\lambda = 0)$ at rocketsonde stations. Finally, it follows that even if the observations are not at the same time, as long as there is a consistent pattern of observations which is similar at each station, the autocorrelation function for some fixed t = constant is available. Now the family of points for $R(t_i,\Delta\lambda_i)$ is available for $t_i = c_1,c_2,\ldots c_n$ and $\Delta\lambda_i = k_1,k_2,\ldots k_m$. It follows a surface could be fitted to the points by a method such as a two-dimensional finite element technique, and this approximate value for $R(t,\lambda)$ could then be used to solve the original equally spaced wavenumber-frequency problem.

CONCLUSIONS

It would first appear that the problems associated with very low density and unequally spaced observations of any meteorological parameters would lead to very limited results. However, many features of the nature of mean and turbulent motion in the stratosphere and lower mesosphere in the tropics can be determined.

An analysis of the mean wind and total variance of the mean wind indicates areas of maximum mean kinetic energy that exist in the tropics. It appears the dominant feature is the late summer and winter easterly jets. This is the area where the total mean kinetic energy is probably dominated by the east-west velocity component, and the east-west component usually exceeds the north-south component by more than one order of magnitude.

Analysis of the turbulent winds and the average seasonal variance and eddy kinetic energy of the turbulent winds indicates the maximum total variance and energy is again assoriated with the east-west velocity component. This is particularly true for the long period seasonal waves which dominate the total energy spectrum. Additionally, there is an energy shift for the east-west component into the longer period waves with altitude increasing from 30 km to 50 km.

While the east-west energy peak is in the 42 to 84 day period, the energy peak for the north-south component is in the 28 to 42 day period. Finally, the synoptic scale waves appear to have a more significant effect on the north-south velocity component. This is most strongly supported by a seasonal variance analysis.

The results of this study verify that the techniques of unequally spaced time series analysis do allow determination of the normalized Eulerian time spectrum. However, the numerical problems encountered, which require analysis of banded frequencies to get stable solutions, make interpretation of the results very difficult.

In the future, with a reasonably large data base, it appears that an estimation of the autocorrelation function and then application of direct Fourier analysis would be a more tenable problem. In fact, visual inspection of the normalized covariances indicate sufficient data may exist at rocketsonde stations to allow estimation of an autocorrelation function based on a period near one day. Finally, the techniques of unequally spaced analysis applied to two dimensions probably are numerically unsolvable without estimating the surface of the two-dimensional autocorrelation function and, in turn, using the fast Fourier transform to estimate the energy spectrum.

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15. Supplementary Notes

16. Abstract

Wind velocities from 25 km to 60 km over Ascension Island, Fort Sherman and Kwajalein for the period January 1970 to December 1971 are analyzed in order to achieve a better understanding of the mean flow, the eddy kinetic energy and the Eulerian time spectra of the eddy kinetic energy.

Since the data is unequally spaced in time, techniques of one-dimensional covariance theory are utilized and an unequally spaced time series analysis is accomplished. The problems in the analysis of unequally spaced time series data are discussed and the theoretical equations for two-dimensional analysis or wavenumber frequency analysis of unequally spaced data are developed.

Analysis of the turbulent winds and the average seasonal variance and eddy kinetic energy of the turbulent winds indicates the maximum total variance and energy is associated with the east-west velocity component. This is particularly true for the long period seasonal waves which dominate the total energy spectrum. Additionally, there is an energy shift for the east-west component into the longer period waves with altitude increasing from 30 km to 50 km.

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